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# Multicriteria decision making for innovation partner selection in a fuzzy environment



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OSSERVATORIO SU SISTEMI  
LOCALI DI INNOVAZIONE





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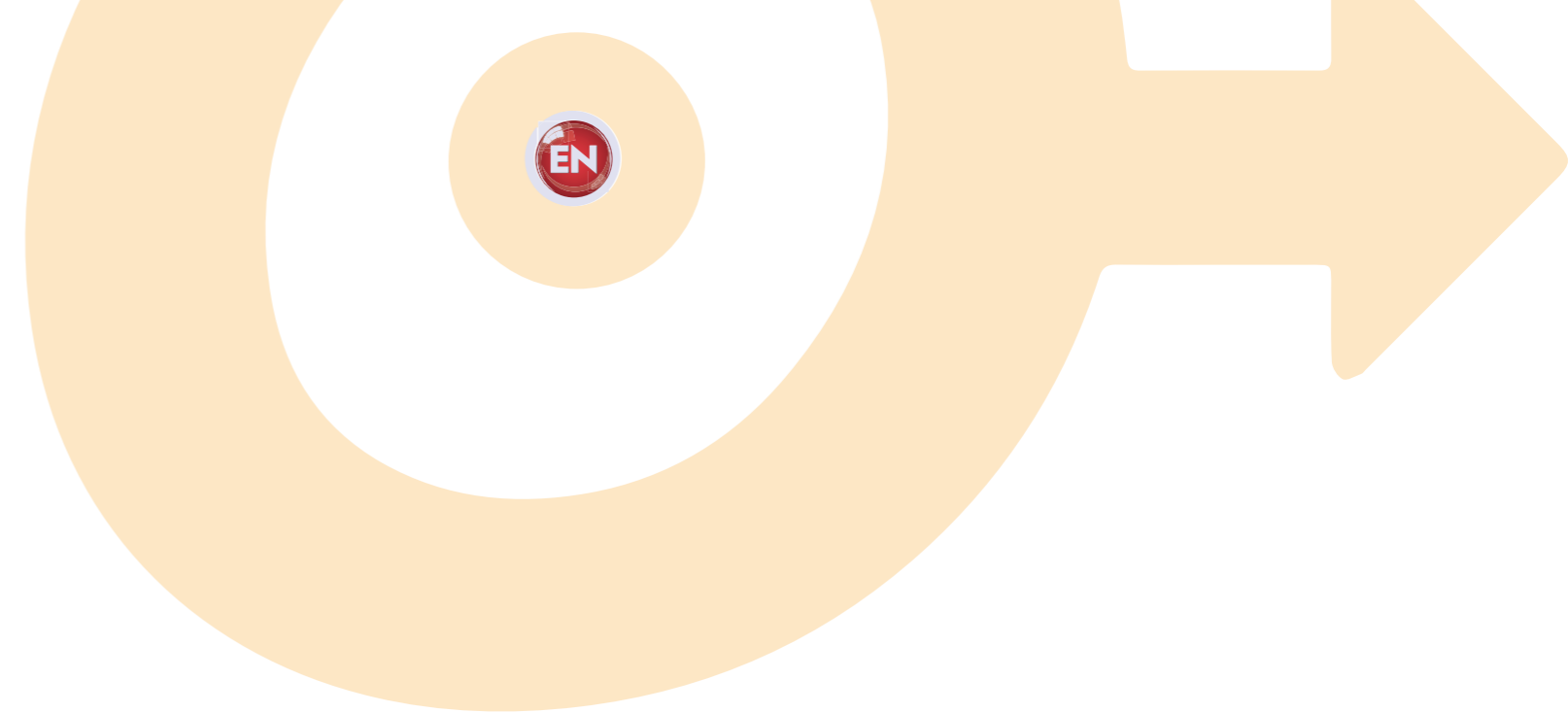
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## Abstract

The quick environment development makes impossible that an enterprise, working individually, can respond to market opportunities. For this reason, more and more firms are aware and motivated to improve their offer and competitiveness by means of *collaboration*, through sharing competencies and resources. In this context, finding the suitable partners is an important issue and the key step in the formation of any collaboration. Further, the decision on which partner should be selected for each task depends, not only on operational characteristics such as cost, quality, etc., but also on financial situation, capability to work in collaboration with other partners and even past performance in collaborative processes. Several researchers have investigated the Partner Selection (PS) criteria in different context. The PS problem is considered as a Multi Criteria Decision Making (MCDM) problem where several evaluation criteria must be considered. Multicriteria decision analysis (MCDA) evaluates options with an additive independent value function. Numerical evaluation criteria (scores) are increasingly difficult to assign, because of inherent complexity of the subject and of imprecise or incomplete knowledge on part of the decision makers.

Aim of the present paper is to give a wider generalization of Multiple Criteria Decision Making (MCDM), for partner selection, where the information is *uncertain*: imprecise /incomplete, both in *evaluations* and *weights*. Fuzzy sets are a natural choice to handle uncertainty of decision makers. In the present work, triangular fuzzy numbers are used and combined with each other by means of the interval algebra operations on their alphacuts. A recently developed method to compute the maximum and minimum between single peaked fuzzy numbers permits to identify the maximum and minimum of aggregated preference scores as reference points to be used for ranking and selection.



Keywords:

**Multiple Criteria Decision Making**

**Partner Selection**

**Fuzzy numbers**

**Fuzzy maximum/minimum**

**Single peaked fuzzy numbers**

# MULTICRITERIA DECISION MAKING IN WA



Digitization is causing a transition with deep consequences on entrepreneurs and firms. Information and knowledge are being produced at an unprecedented rate and the success of business initiatives largely depends on how these factors and how efficiently they are managed (Carayannis et al., 2006).



## 1. Introduction

### 1. Introduction

Digitization is causing a transition with deep consequences on entrepreneurs and firms. Information and knowledge are being produced at an unprecedented rate and the success of business initiatives largely depends on how these factors and how efficiently they are managed (Carayannis et al., 2006). New technologies are introduced, but this process is gradual, creating the need to manage digital and legacy production plants, as well as innovative and traditional business models. The ability to adapt to technological changes quickly is a powerful factor for fruitful cooperation within Innovation Systems (ISs), both at an intraorganizational level and at an interorganizational level. Postponing the adoption emerging technology until it is mature may result in a competitive disadvantage, because adopting innovative technology early emerging technology can also bring substantial returns (see, e.g., (Castellano et al., 2019) for a recent analysis). However, investments in innovative technology when it is in its early stages is risky. To pioneer new technologies that are not their own development, firms need to search for, acquire and integrate knowledge from the outside, often the academic community. Through a meaningful mutual exchange between industry and academia, firms can assimilate external knowledge and researchers can concentrate their efforts on realworld problems. Industryacademia collaboration has been the subject of rich literature on technology transfer and corporate strategic technology alliances (Carayannis et al., 2014). Such collaboration can result in incubators associated with universities, offering firms businesses support services as well as assistance in seeking financial backing (Carayannis et al., 2006). Technology clusters, supported by a reliable and efficient ICT infrastructure, enable firms to thrive on synergies deriving from proximity (Carayannis et al., 2006). Interaction between industry and academia can also happen when employees publish their research results and present them in conferences (Tegarden et al., 2011).

A firm can acquire technological knowledge in different ways. Firm can invest in their internal R&D departments, outsource R&D with third parties, or buy firms who already have developed the desired assets. While internal R&D may be attractive, it adds uncertainty because firms do not know in advance what the innovative output is going to be. In addition, it takes time to set up a productive team: the most effective setting involves the creation of a knowledgecentered organizations, where learning is encouraged and supported. Such organizations are attractive for employees and increase their level of satisfaction (Janz and Prasarnphanich, 2009). As Intellectual Capital becomes more important, motivation of employees accentuates its prominence because, when dissatisfied workers leave an organization,

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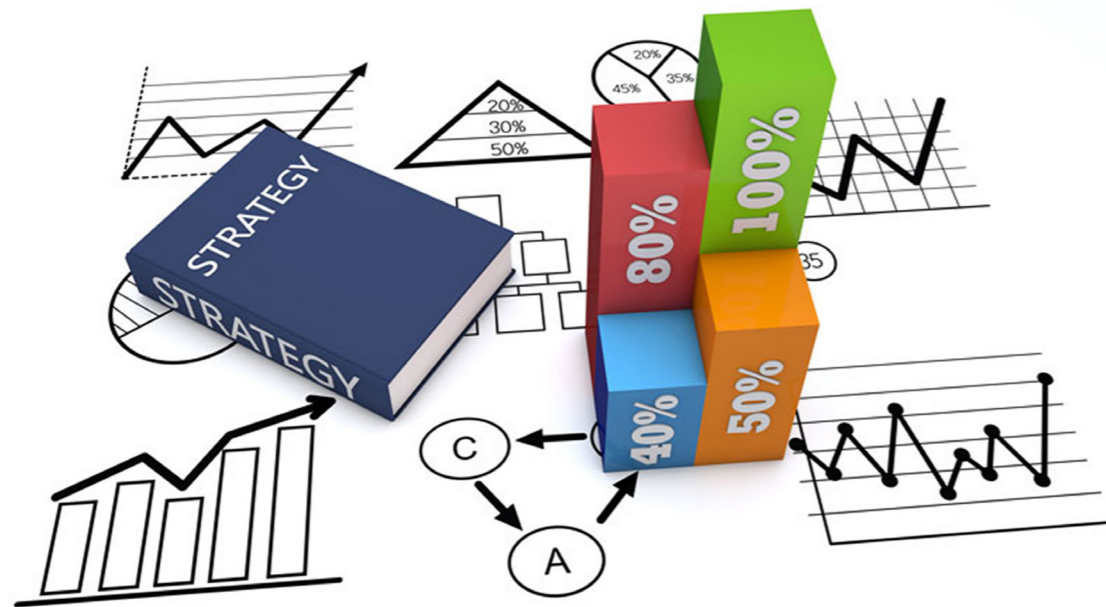
their knowledge, fully integrated in the firm and immediately applicable, leaves with them (Nicolaescu et al., 2019). Even in firms where the core internal competency is technological development, outsourcing at least some R&D can be beneficial (Cefis and Triguero, 2016). In the first place, a firm has to decide whether or not to perform R&D internally. Once a firm has determined to look for partners, contracting R&D with external entities involves an accurate search for reliable and capable partners. Further costs arise from the need to prepare robust contracts. Besides partnerships, mergers and acquisitions (M&A) can also be considered (Cefis and Triguero, 2016). While M&A requires significant financial and organizational effort, it may bring the additional benefit of improving efficiency (economies of scale and sharing R&D expenditures) and even correcting managerial inefficiencies and weak corporate control mechanisms. All of these decisions involve an articulated assessment of costs and benefits, and the available information is almost always characterized by vagueness, ambiguity, and uncertainty. There are many methods in the field of decision analysis that try to help a decision maker take a decision, that is, to find an optimal or satisfying solution. Multicriteria decision analysis (MCDA) evaluates options with an additive independent value function (Pape, 2017).

The aim of this paper is to give a wider generalization of Multiple Criteria Decision Making (MCDM) where information is incomplete about evaluations *and* weights in the framework of *fuzzy numbers*. A fuzzy linear programming technique for solving Multiattribute Decision Making (MADM) problems with multiple types of attribute values and partial information about weights has been proposed in (Li and Wan, 2013), using trapezoidal fuzzy numbers. The MCDM has been generalized to the case of *interval judgments on the weights* and *interval value scores* (Mustajoki et al., 2005) in order to consider preferential uncertainty, imprecision, or incomplete information. The SMART and SWING methods for point estimates are generalized using interval judgements to reflect imprecision. In SMART, the user begins with identifying the least important criterion, and then the remaining criteria are rated relative to the least important one. In SWING, decision makers are iteratively asked to identify the criterion where they would most prefer a change from the worst value to the best value. The reference attribute is crisp, and subsequent assessments are expressed with interval values, resulting in constraints that determine the feasible region for weights. A decision support framework for planning and management in electrical energy companies, offering a variety of fuzzy preference relations has been proposed by Koshenev *et al.* (Kokshenev et al., 2014) In the context of selecting the best policy alternative on the energy market, Kahraman and Kaya (Kahraman and Kaya, 2010) applied a modified fuzzy Analytic Hierarchy

## 1. Introduction

Process (AHP) method to compute the priority weights of energy policy alternatives. In that work, a method based on distances has been used for ranking the fuzzy scores in order to select the best alternatives. However, the reference targets were the crisp maximum and minimum. In contrast, the fuzzy maximum and minimum have been used here. The calculation of fuzzy minimum and maximum plays an important role because the weighted sum of fuzzy evaluation scores when the weight themselves are fuzzy numbers produces fuzzy numbers of any shape.

The remainder of this manuscript is structured as follows. After a brief review of the vast related literature in Section 2, essential concepts and notations are described in Section 3. The proposed technique is elucidated in Section 4. Finally, Section 5 contains some final remarks and outlines directions for future research.



## 2. Related work

### 2. Related Work

The problem of green supplier evaluation and selection has been investigated in (Qin et al., 2017), where type2 fuzzy sets and prospect theory are combined into an integrated approach Multiple Criteria Group Decision Making (MCGDM). Consistency for interval fuzzy preference relations (IFPRs) is receiving considerable attention from the scientific community (Meng et al., 2019). In the AHP model, developed by Saaty (Saaty, 1980), a decision-making problem is hierarchically decomposed into a series of choices involving alternatives and preference criteria. The best solution is obtained through pairwise comparisons of alternatives according to criteria, giving rise to a collection of preference relations. Within the context of fuzzy AHP, intervalvalued matrices can be used to describe uncertainty on part of decision makers (Pape, 2017). To add further complexity, the interval-valued comparison matrix may be incomplete, expressing lack of knowledge by decision makers. The estimation of missing entries in the incomplete intervalvalued matrices can be done by means of goal programming models (Huang et al., 2019), based on appropriate redefinition of multiplicative and additive approximate consistency rules for interval additive reciprocal preference relations (IARPRs).

In portfolio decision analysis, more than one option can be selected. Interesting ideas have been developed in the context of investment portfolio selection problems. In such problems, investors make choices based on their personal knowledge and experience, because quantities like risk, expected returns, and liquidity are subject to uncertainty and, as a consequence, deterministic algorithms are not effectively applicable. Given the presence of nonstochastic elements in the market, fuzzy models have been proposed for portfolio selection. A strategy for attacking the portfolio selection problem involves first transforming the fuzzy optimization problem into a multi-objective optimization problem, and then solving the latter by means of fuzzy decision-making techniques (Solatikia et al., 2014). Investor preferences determine the optimal multi-objective solution according to alternative scenarios. A fuzzy multiobjective model for portfolio selection, striking a balance between strategic contributions and financial returns, was proposed by Guo et al. (Guo et al., 2018). The solution was obtained by the adoption of a MultiObjective Genetic Algorithm.

Relich and Pawlewski (Relich and Pawlewski, 2017), on the basis of the observation that the criteria for product selection are often uncertain and complex, combined neural networks and fuzzy weighted average for project ranking. Individual weights about the importance of each product were gathered by means of a questionnaire, transforming linguistic variables into fuzzy numbers. The selection of the products having the highest fuzzy



## 2. Related work

score while at the same time meeting manufacturing constraints was done through a neural network.

Finally, to support strategic decisions about the management and planning of clinical trials in the pharmaceutical industry and provide managers at different responsibility levels with information concerning financial and operational risks, three structured fuzzy inference systems (FISs) were used (Puentes et al., 2019)



## 3. Preliminaries

### 3. Preliminaries

In this section some definitions and basic results from the theory of fuzzy sets are reported.

Wide surveys and references may be found in (Zadeh, 1965), (Zimmermann, 2011), (Klir and Yuan, 1995). Let  $X$  be a universal set, with a generic element of  $X$  denoted by  $x$ . Thus,  $X = \{x\}$ . A fuzzy set  $N$  in  $X$  (fuzzy subset  $N$  of  $X$ ) is characterized by a membership function  $\mu_N(x)$  which associates with each point in  $X$  a real number in the interval  $[0,1]$ . The value  $\mu_N(x)$  represents the grade of membership of  $x$  in  $N$ . When  $N$  is a set in the ordinary sense of the term, its membership can take only two values 0 and 1 according as  $x$  does or does not belong to  $N$ . Let  $N$  be a fuzzy subset of  $X$ , the  $\alpha$ -cut of  $N$  is defined as:

$$N[\alpha] = \{x \in X \mid \mu_N(x) \geq \alpha\}, \text{ for } 0 < \alpha \leq 1$$

The core of  $N$ ,  $co(N)$ , is  $N[1]$  while the support of  $N$ ,  $sp(N)$ , of  $N$  is not  $N[0]$  (which is always the whole universe  $X$ ) but  $sp(N) = \{x \in X \mid \mu_N(x) > 0\}$ . With  $\overline{sp}(N)$  will be denoted the closure of the support of  $N$ . A fuzzy number is called *single peak* if the core is a singleton.

Referring to (Goetschel Jr and Voxman, 1986) (see also (Chai and Zhang, 2016)) a fuzzy number  $N$  is a fuzzy subset of  $\mathbb{R}$ , with membership function denoted by  $\mu_N$ , satisfying the following conditions:

1.  $\mu_N$  is normal, i.e.,  $co(N)$  is non-empty.
2.  $\mu_N$  is compactly supported, i.e.,  $\overline{sp}(N)$  is bounded;
3.  $\mu_N$  is quasi-concave, i.e.,  $x \leq y \leq z$  implies:  

$$\min \{\mu_N(x), \mu_N(z)\} \leq \mu_N(y) \quad \forall x, y, z \in \mathbb{R};$$
4.  $\mu_N$  is upper semi-continuous, i.e.,  $\forall \alpha \in ]0,1]$ ,  $N[\alpha]$  is closed.

In (Goetschel Jr and Voxman, 1986) the following characterization of a fuzzy number is given.

Defining

$$N[\alpha] = \begin{cases} \{x \in \mathbb{R} \mid \mu_N(x) \geq \alpha\}, & \text{if } 0 < \alpha \leq 1; \\ \overline{sp}(N), & \text{if } \alpha = 0; \end{cases} \quad (1)$$

### 3. Preliminaries

it can be shown that  $N$  is a fuzzy number if and only if

1.  $N[\alpha]$  is a closed and bounded interval  $\forall \alpha \in [0,1]$ ;
2.  $N[1] \neq \emptyset$

Using this characterization, we can identify a fuzzy number  $N$  with the parameterized representation  $\{(N[\alpha], \bar{N}[\alpha]) \mid \alpha \in [0,1]\}$ , where  $\bar{N}[\alpha]$  and  $N[\alpha]$  denote respectively the left hand endpoint and the right hand endpoint of  $N[\alpha]$ . By this parameterized representation, given two fuzzy numbers  $N, M$  and a real number  $c$ , these may be combined by means of the interval algebra operations as reported below:

1.  $N + M = \{(N[\alpha] + M[\alpha], \bar{N}[\alpha] + \bar{M}[\alpha]) \mid \alpha \in [0, 1]\}$
2.  $N - M = \{(N[\alpha] - \bar{M}[\alpha], \bar{N}[\alpha] - M[\alpha]) \mid \alpha \in [0, 1]\}$
3.  $N \cdot M = \{(P[\alpha], \bar{P}[\alpha]) \mid \alpha \in [0, 1]\}$ ,

where

$$\begin{aligned} P[\alpha] &= \min\{N[\alpha]M[\alpha], N[\alpha]\bar{M}[\alpha], \bar{N}[\alpha]M[\alpha], \bar{N}[\alpha]\bar{M}[\alpha]\} \\ \bar{P}[\alpha] &= \max\{N[\alpha]M[\alpha], N[\alpha]\bar{M}[\alpha], \bar{N}[\alpha]M[\alpha], \bar{N}[\alpha]\bar{M}[\alpha]\} \end{aligned}$$

4. If  $\forall \alpha \in [0, 1], 0 \notin M[\alpha]$

$$\frac{N}{M} = \left\{ \left( \frac{N[\alpha]}{M[\alpha]}, \frac{\bar{N}[\alpha]}{\bar{M}[\alpha]} \right) \mid \alpha \in [0, 1] \right\}$$

5.  $cN = \begin{cases} \{(cN[\alpha], c\bar{N}[\alpha]) \mid \alpha \in [0, 1]\}, & \text{if } c > 0; \\ \{(c\bar{N}[\alpha], cN[\alpha]) \mid \alpha \in [0, 1]\}, & \text{if } c < 0; \\ 0, & \text{if } c = 0. \end{cases}$

It is known that, in many practical applications when handling with fuzzy numbers, it is necessary to have a permanent switch from a fuzzy representation to a numerical one. This transformation is usually carried out by the defuzzification process which, however, may cause loss of information. By means of the parameterized representation already described, fuzzy numbers are combined one to another making use of the *interval algebra* instruments preserving all the information in the data. Operations between fuzzy numbers are easily implemented on a computer by means of interval arithmetic on their  $\alpha$ -cuts, which are always closed and bounded intervals for  $0 \leq \alpha \leq 1$ . The  $\alpha$ -cuts of a fuzzy number are univocally determined computing left/right side membership inverse.

### 3. Preliminaries

#### 3.1. Fuzzy maximum and minimum

A fuzzy number  $A$  is called *single peak* if  $co(A)$  is a singleton  $\{x_0\} \in \mathbb{R}$  called the *mean value* of  $A$ . It is important to remark that if  $N$  and  $M$  are single peak fuzzy numbers and  $c \in \mathbb{R}$ , also  $cN$  and  $N \cdot M$ , with  $\cdot \in \{+, -, \cdot, \div\}$ , are single peak fuzzy numbers. Very popular single peak fuzzy numbers are *triangular fuzzy numbers*. Denote with  $\underline{N} = \inf sp(N)$ ,  $\bar{N} = \sup sp(N)$  and with  $\hat{N} = co(N)$  then the triangular fuzzy number is denoted by  $N = (\underline{N}, \hat{N}, \bar{N})$ , while its membership function is defined as follows

$$\mu_N(x) = \begin{cases} \frac{x - \underline{N}}{\hat{N} - \underline{N}} & \text{if } \underline{N} \leq x \leq \hat{N}; \\ \frac{x - \bar{N}}{\hat{N} - \bar{N}} & \text{if } \hat{N} < x \leq \bar{N}; \\ 0, & \text{otherwise.} \end{cases}$$

It is important to remark that traditional mathematics is based on *crisp* set theory. Fuzzy mathematics is based on fuzzy set theory. Traditional mathematics becomes a special case of fuzzy mathematics when all membership functions are restricted to have values only zero and one. Each real number  $c$  can be identified with the crisp fuzzy number having membership equal to

$$\mu_c(x) = \begin{cases} 1 & \text{if } x = c; \\ 0, & \text{otherwise.} \end{cases}$$

so, in particular, each real number can be seen as a fuzzy number having single peak.

While we are familiar with the computation of the minimum and the maximum between real numbers, we are not familiar with that computation between fuzzy numbers instead. Following (Buckley and Eslami, 2002), given two continuous fuzzy numbers  $N$  and  $M$ , if  $P = \max(N, M)$  and  $Q = \min(N, M)$  then the membership functions of  $P$  and  $Q$  are respectively:

$$\mu_P(z) = \sup\{\min\{\mu_M(x), \mu_N(y)\} \mid \max\{x, y\} = z\}$$

and

$$\mu_Q(z) = \sup\{\min\{\mu_M(x), \mu_N(y)\} \mid \min\{x, y\} = z\}.$$



### 3. Preliminaries

If  $N$  and  $M$  are single peak fuzzy numbers, as proved in (Hong and Kim, 2006) (see also (Chiu and Wang, 2002)), the  $\alpha$ -cut of  $P$  and  $Q$  are given by

$$P[\alpha] = [\max\{\underline{N}[\alpha], \underline{M}[\alpha]\}, \max\{\overline{N}[\alpha], \overline{M}[\alpha]\}]$$

and

$$Q[\alpha] = [\min\{\underline{N}[\alpha], \underline{M}[\alpha]\}, \min\{\overline{N}[\alpha], \overline{M}[\alpha]\}]$$

thus  $P$  and  $Q$  have the following parameterized representations

$$P = \{(\max\{\underline{N}[\alpha], \underline{M}[\alpha]\}, \max\{\overline{N}[\alpha], \overline{M}[\alpha]\}) \mid \alpha \in [0, 1]\}$$

and

$$Q = \{(\min\{\underline{N}[\alpha], \underline{M}[\alpha]\}, \min\{\overline{N}[\alpha], \overline{M}[\alpha]\}) \mid \alpha \in [0, 1]\}$$

### 4. The Method

#### 4. The Method

Let us consider a set of options  $I = \{1, \dots, n\}$  and a set of criteria  $J = \{1, \dots, k\}$ . The performance of each option  $i \in I$  is valued with respect to each criterion  $j \in J$  along a certain scale to obtain the value score  $v_{ij}$ . The criteria are weighted generating the relative importance judgements  $w_j$  so that so that the following normalization condition holds true

$$\sum_{j=1}^k w_j = 1. \quad (2)$$

The overall score for option  $i$  is

$$V_i = \sum_{j=1}^k w_j v_{ij} \quad (3)$$

so the decision model suggests to choose the option with highest score  $V_i$  ( $i = 1, \dots, n$ ).

By decision-making in a fuzzy environment, we mean a decision process in which the goals and/or the constraints, but not necessarily the system under control, are constitute classes of alternatives whose boundaries are not sharply defined. Let us suppose that both the value scores  $v_{ij}$  and the weights  $w_j$  ( $i = 1, \dots, n; j = 1, \dots, k$ ) are *uncertain* in the sense that the decision maker is not sure about their exact value but has the idea that, for example, the value score of option  $i$  with respect to criteria  $j$  is *approximately*  $v_{ij}$  and that he wants to assign to criteria  $j$  a weight that is *approximately*  $w_j$ . This kind of uncertain information may be modelled assuming the above uncertain quantities to be *single peak fuzzy numbers* given by the following parameterized representation:

$$\begin{aligned} & \{(\underline{v}_{ij}[\alpha], \overline{v}_{ij}[\alpha]) \mid 0 \leq \alpha \leq 1\} \\ & \{(\underline{w}_j[\alpha], \overline{w}_j[\alpha]) \mid 0 \leq \alpha \leq 1\} \end{aligned}$$

for  $i = 1, \dots, n$  and  $j = 1, \dots, k$ .

#### 4. The Method

The fuzzy overall score of option  $i$

$$\{(\underline{V}_i[\alpha], \bar{V}_i[\alpha]) \mid 0 \leq \alpha \leq 1\} \quad (4)$$

is computed from (3) by the interval algebra operations as follows:

$$\underline{V}_i[\alpha] = \sum_{j=1}^k \underline{P}_j[\alpha] \quad (5)$$

and

$$\bar{V}_i[\alpha] = \sum_{j=1}^k \bar{P}_j[\alpha] \quad (6)$$

where  $\underline{P}_j[\alpha]$  and  $\bar{P}_j[\alpha]$  are respectively

$$\min\{\underline{w}_j[\alpha]v_{ij}[\alpha], \underline{w}_j[\alpha]\bar{v}_{ij}[\alpha], \bar{w}_j[\alpha]v_{ij}[\alpha], \bar{w}_j[\alpha]\bar{v}_{ij}[\alpha]\}$$

$$\max\{\underline{w}_j[\alpha]v_{ij}[\alpha], \underline{w}_j[\alpha]\bar{v}_{ij}[\alpha], \bar{w}_j[\alpha]v_{ij}[\alpha], \bar{w}_j[\alpha]\bar{v}_{ij}[\alpha]\}$$

The normalization condition (2) is retrieved inasmuch as it holds on the mean values of the fuzzy numbers  $\{(\underline{w}_j[\alpha], \bar{w}_j[\alpha]) \mid 0 \leq \alpha \leq 1\}$  for  $j = 1, \dots, k$ .

It is important to remark that, while the input data  $(v_{ij}, w_j, i = 1, \dots, n; j = 1, \dots, k)$  of the introduced model are triangular fuzzy numbers, the fuzzy overall score (4) it is not. It is known in fact that, if  $M$  and  $N$  are triangular fuzzy numbers, then so is  $M + N$  and  $M - N$ , however  $M \cdot N$  will be a triangular shaped fuzzy number.



#### 4. The Method

Once the overall fuzzy scores are computed for each option, the decision model should recommend the option with the *greatest* fuzzy score (4). It is known however that, given two fuzzy numbers  $M$  and  $N$ ,  $P = \max(M, N)$  and  $Q = \min(M, N)$  not always equals  $M$  or  $N$ . A methodology for ranking fuzzy numbers has been necessary for this purpose.

Ranking fuzzy numbers is the subject of a vast literature (Asady and Zendehnam, 2007). In this work, a combination of the methods proposed by Tran and Duckstein (Tran and Duckstein, 2002) and Asady and Zendehnam (Asady and Zendehnam, 2007) is used for ranking fuzzy scores. The method is based on the comparison of distances from fuzzy numbers (FNs) to some predetermined targets.

For arbitrary fuzzy numbers in parametric form  $N$  and  $M$ , the quantity

$$D(N, M) = \sqrt{\int_0^1 (\underline{N}[\alpha] - \underline{M}[\alpha])^2 d\alpha + \int_0^1 (\bar{N}[\alpha] - \bar{M}[\alpha])^2 d\alpha}$$

is the distance between  $N$  and  $M$  (Asady and Zendehnam, 2007).

A fuzzy number will be ranked first if its distance from the maximum  $M$  is the smallest and its distance from the minimum  $m$  is the greatest. When only one of the previous conditions is satisfied, ranking should be determined by the attitude of the decision maker towards risk. A risk-averse decision maker might prefer a score that is as far as possible from the minimum, whereas a risk seeking individual would rather choose the closest score to the maximum. The reader interested in this subject is referred to the work of Yu *et al.* (Yu *et al.*, 2018) where, in essence, compromise-typed variable weight functions are constructed by means of utility functions and the variable weight decision making is done through the inspection of variable weight synthesis and the *orness* measures derived from the coefficients of absolute risk aversion.



## 4. The Method

### 4.1. Technical Considerations

Let us remark that to extend the MCDM to a *fuzzy framework* it has been necessary to consider:

an algebra over the set of fuzzy numbers;  
a method for computing  $\min / \max\{N, M\}$  for single peak fuzzy numbers  $N, M$ .

1. It is known that, in many practical applications when handling with fuzzy numbers, it is necessary to have a permanent switch from a fuzzy representation to a numerical one. This transformation is usually carried out by the defuzzification process which, however, may cause loss of information. In this paper fuzzy numbers are combined one to another without any defuzzification method but making use of the *interval algebra* instruments. Operations between fuzzy numbers can be easily implemented on a computer by means of interval arithmetic on their  $\alpha$ -cuts, which are always closed and bounded intervals for  $0 \leq \alpha \leq 1$ . The  $\alpha$ -cuts of a fuzzy number are univocally determined computing left/right side membership inverse.

2. While we are familiar with the computation of the minimum and the maximum between real numbers, we are not familiar with that computation between fuzzy numbers instead. In this paper we follow the approach of (De Marco et al., 2020) which present and implement a new method for computing

$$\min\{N, M\}, \max\{N, M\}$$

where  $N$  is a triangular fuzzy number while  $M$  is a fuzzy number single peak of any shape. A little discernment with respect to (De Marco et al., 2020) has been introduced in the case  $N$  and  $M$  are both single peak of any shape.

## 5. Conclusion

### 5. Conclusion

In this work, the MCDM problem with imprecision in both the evaluations and the criteria weights is addressed. For each of the considered criteria, a decision maker evaluates the convenience of an alternative from a personalized viewpoint, resulting in an imprecise score which can be modelled by a triangular fuzzy number. The overall score of the alternative is then determined by aggregating the preference scores according to weights that quantify their importance, and imprecision in these weights can again be expressed with triangular fuzzy numbers. The resulting weighted sum is no longer a triangular fuzzy number, but it can be a fuzzy number of any shape. By computing the fuzzy minimum and maximum of the scores and using them as reference values, alternatives can be ranked. The advantage of the proposed method is that it is contained in the framework of fuzzy numbers. Directions for future development include the extension of the proposed method to the MCGDM, where a group of decision makers need to integrate their preferences. If an aggregation approach is chosen based on weights reflecting the relative importance of individual decision makers (perhaps related to their hierarchical position or their perceived knowledge) and such weights are fuzzy, a method that is able to handle the results of individual preferences and the details about their ranking will be needed.





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